

On augmented superfield approach to vector Schwinger model

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Abstract: We exploit the techniques of Bonora-Tonin superfield formalism to derive the off-shell nilpotent and absolutely anticommuting (anti-)BRST as well as (anti-)co-BRST symmetry transformations for the $(1 + 1)$ -dimensional (2D) bosonized vector Schwinger model. In the derivation of above symmetries, we invoke the (dual)-horizontalness conditions as well as gauge and (anti-)co-BRST invariant restrictions on the superfields that are defined onto the $(2, 2)$ -dimensional supermanifold. We provide geometrical interpretation of the above nilpotent symmetries (and their corresponding charges). We also express the nilpotency and absolute anticommutativity of the (anti-)BRST and (anti-)co-BRST charges within the framework of augmented superfield formalism.

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1 Introduction

In order to covariantly quantize a gauge theory, Becchi-Rouet-Stora-Tyutin (BRST) formalism provides one of the most natural frameworks. In BRST formalism, the unitarity and “quantum” gauge (i.e. BRST) invariance are respected together at any arbitrary order of perturbative computations [1–4]. The BRST formalism has found its application in many of the modern theoretical developments in the area of quantum field theories and superstring theories. Recently, the BRST approach has been applied to construct a Lagrangian for the fermionic higher spin fields [5] as well as in the study of massless and massive fields with totally symmetric arbitrary spin [6] in AdS space. Moreover, BRST quantization of the pure spinor superstring has been carried out [7] and its cohomological aspects have also been established in the pure spinor formalism [8].

There are two pivotal properties associated with the (anti-)BRST symmetries (and their corresponding charges): (i) nilpotency of order two, and (ii) anticommutativity. The origin and geometrical interpretation of these abstract mathematical properties are provided by the superfield approach to the BRST formalism (see Refs. 9–13). In this formalism, a given D -dimensional gauge theory is generalized onto a $(D, 2)$ -dimensional supermanifold characterized by a pair of the Grassmannian superspace coordinates $(\theta, \bar{\theta})$. The horizontality condition (HC) and gauge invariant restrictions (GIRs) play a central role in the derivation of the (anti-)BRST symmetry transformations for the gauge and corresponding (anti-)ghost fields for a underlying gauge theory. Furthermore, the geometrical basis for the (anti-)BRST symmetries and corresponding generators is provided by the translational generators along the Grassmannian directions.

In our present study, we apply extensively the above mentioned geometrical superfield formalism to discuss various symmetry properties for the bosonized vector Schwinger model (VSM) within the framework of BRST formalism. The VSM, an offshoot of Schwinger model – quantum electrodynamics in $(1 + 1)$ -dimensions with massless fermions, is a well-known model in the regime of two-dimensional field theories [14–23]. It is a gauge invariant and exactly solvable model that is endowed with the first-class constraints in the language of Dirac’s prescription for the classification of constrained systems. This model has been studied within the framework of BRST as well as Hamiltonian formalism [24]. In a most recent development, it has been shown that the VSM is equipped with, in totally, *six* continuous symmetries, namely, (anti-)BRST symmetries, (anti-)co-BRST symmetries, bosonic symmetry and a ghost scale symmetry [25]. This makes VSM to be a tractable model for the Hodge theory [26–29].

Our present investigation is essential on the following ground. First, to derive the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations in a physically intuitive manner by exploiting the power and strength of the HC and GIRs (within the framework of superfield formalism). Second, to provide the geometrical origin of the above mentioned symmetries (and their corresponding generators) in the language of translational generators along the Grassmannian directions of the $(2, 2)$ -dimensional supermanifold, on which, VSM is generalized.

The structure of the paper is organized as follows. In Sec. 2, we briefly recapitulate the constraint structure and gauge symmetry for the $(1 + 1)$ -dimensional (2D) bosonized version of VSM. We derive the off-shell nilpotent and absolutely anticommuting (anti-

)BRST symmetries for VSM, within the framework of augmented superfield formalism, in our Sec. 3. Section 4 is devoted to the derivation of nilpotent and absolutely anticommuting (anti-)co-BRST symmetries and their geometrical interpretation. In Sec. 5, we capture the nilpotency and absolute anticommutativity properties of the (anti-)BRST as well as (anti-)co-BRST charges within the framework of superfield formalism. We also provide the geometrical origin of these mathematical properties in this section. Finally, in Sec. 6, we make some concluding remarks. In Appendix, we provide precise proof for the *ad hoc* choice of auxiliary field that being made in the Sec. 4.

2 Preliminaries: constraint structure and gauge symmetries

We begin with the following Lagrangian density of 2D bosonized version of VSM:^{*} [18, 25]

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - e\varepsilon^{\mu\nu}\partial_\mu\phi A_\nu \\ &\equiv \frac{1}{2}E^2 + \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi'^2 + e\dot{\phi}A_1 - e\phi'A_0,\end{aligned}\tag{1}$$

where the overdot and prime on the fields denote the time and space derivatives, respectively. In 2D, the curvature tensor $F_{\mu\nu}(= \partial_\mu A_\nu - \partial_\nu A_\mu)$ has only electric field E as its existing component and the gauge field A_μ is topological in nature (in the case of 2D). In the above, ϕ is massless bosonic field and e represents electric charge as the coupling constant.

The above Lagrangian density is endowed with two first-class constraints: $\Pi^0 \approx 0$ and $\partial_1\Pi^1 - e\partial_1\phi \approx 0$ in Dirac's terminology [25]. Here, Π^0 and $\Pi^1(= -F^{01} = E)$ are the canonical conjugate momenta with respect to the fields A_0 and A_1 , respectively. The canonical conjugate momentum with respect to ϕ is $\Pi_{(\phi)} = \dot{\phi} + eA_1$. We work with the first-order Lagrangian density which can be written as follows:

$$\mathcal{L}_f = \frac{1}{2}\left(E^2 - \Pi_\phi^2 - \phi'^2 - e^2 A_1^2\right) + \Pi_\phi \dot{\phi} + e \Pi_\phi A_1 - e \phi' A_0.\tag{2}$$

It is well known fact that the presence of first-class constraints indicates VSM to be a gauge field theoretic model. The most general form of the generator (G) in terms of first-class constraints, which generates the gauge transformations, can be given as

$$G = \int dx \left[\dot{\chi} \Pi^0 - \chi (E' - e\phi') \right],\tag{3}$$

where $\chi(x, t)$ is an infinitesimal local gauge parameter. The local gauge symmetries generated from the above generator are given as follows:

$$\delta A_0 = \dot{\chi}, \quad \delta A_1 = \chi', \quad \delta \Pi_\phi = e\chi', \quad \delta \phi = 0, \quad \delta E = 0.\tag{4}$$

It can be readily checked that the action integral remains invariant under the above local gauge transformations.

^{*}Here, we choose the 2D flat metric $\eta_{\mu\nu}$ with signature $(+1, -1)$ with the Greek indices $\mu, \nu \dots = 0, 1$. The 2D Levi-Civita tensor $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$ is such that $\epsilon_{01} = +1 = -\epsilon^{01}$ and it obeys $\epsilon^{\mu\nu}\epsilon_{\mu\nu} = -2!$, $\epsilon^{\mu\nu}\epsilon_{\mu\lambda} = -\delta_\lambda^\nu$, etc.

3 Off-shell nilpotent (anti-)BRST symmetries: superfield approach

We exploit the standard tools and techniques of Bonora-Tonin's (BT) superfield formalism [9, 10] to derive the off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetries for the 2D bosonized version of VSM. Before going into the details of superfield formalism, it is worthwhile to mention that the fields $A_0(x, t)$ and $A_1(x, t)$ are the functions of spacetime variables (x, t) . In the physical 2D of spacetime, we define the exterior derivative (d) and one-form connection ($A^{(1)}$) as follows:

$$d = dt\partial_t + dx\partial_x, \quad A^{(1)} = dtA_0 + dxA_1. \quad (5)$$

In the BT superfield formalism, we generalize the exterior derivative and one-form connection to the super exterior derivative (\tilde{d}) and super one-form connection ($\tilde{A}^{(1)}$) in the superspace which is characterized by, in addition to (x, t) , a pair of Grassmannian variables $\theta, \bar{\theta}$ (with $\theta^2 = \bar{\theta}^2 = 0$, $\theta\bar{\theta} + \bar{\theta}\theta = 0$) as

$$\begin{aligned} d \rightarrow \tilde{d} &= dt\partial_t + dx\partial_x + d\theta\partial_\theta + d\bar{\theta}\partial_{\bar{\theta}}, \\ A^{(1)} \rightarrow \tilde{A}^{(1)} &= dt\tilde{A}_0 + dx\tilde{A}_1 + d\theta\tilde{\mathcal{F}} + d\bar{\theta}\tilde{\mathcal{F}}, \end{aligned} \quad (6)$$

where \tilde{A}_0, \tilde{A}_1 are the superfields corresponding to A_0, A_1 , respectively and $(\tilde{\mathcal{F}})\mathcal{F}$ are the superfields corresponding to the (anti-)ghost fields $(\bar{C})C$. Now these superfields are expanded, in terms of basic and secondary fields of the theory, along the Grassmannian directions in the following fashion:

$$\begin{aligned} \tilde{A}_0(x, t, \theta, \bar{\theta}) &= A_0(x, t) + \theta\bar{f}_1(x, t) + \bar{\theta}f_1(x, t) + i\theta\bar{\theta}B_1(x, t), \\ \tilde{A}_1(x, t, \theta, \bar{\theta}) &= A_1(x, t) + \theta\bar{f}_2(x, t) + \bar{\theta}f_2(x, t) + i\theta\bar{\theta}B_2(x, t), \\ \mathcal{F}(x, t, \theta, \bar{\theta}) &= C(x, t) + i\theta\bar{b}_1(x, t) + i\bar{\theta}b_1(x, t) + i\theta\bar{\theta}s_1(x, t), \\ \tilde{\mathcal{F}}(x, t, \theta, \bar{\theta}) &= \bar{C}(x, t) + i\theta\bar{b}_2(x, t) + i\bar{\theta}b_2(x, t) + i\theta\bar{\theta}s_2(x, t). \end{aligned} \quad (7)$$

In the above expression, $B_1, B_2, b_1, \bar{b}_1, b_2, \bar{b}_2$ are bosonic secondary fields whereas the secondary fields $f_1, \bar{f}_1, f_2, \bar{f}_2, s_1, s_2$ are fermionic in nature.

Now, applying the standard technique of horizontality condition (HC) which imposes the following restriction:

$$dA^{(1)} = \tilde{d}\tilde{A}^{(1)}. \quad (8)$$

In other words, the above condition implies that a 'physical' quantity (i.e. $E(= dA^{(1)})$ in the present case) must remain unaffected by the presence of Grassmannian variables when the former is generalized onto $(2, 2)$ -dimensional supermanifold. Exploiting the above HC (8), we obtain the following algebraic relationships amongst the basic and secondary fields of the theory:

$$\begin{aligned} b_1 &= 0, \quad \bar{b}_2 = 0, \quad s = 0, \quad s_1 = 0, \quad b_2 + \bar{b}_1 = b, \\ f_1 &= \dot{C}, \quad \bar{f}_1 = \dot{\bar{C}}, \quad f_2 = \partial_x C, \quad \bar{f}_2 = \partial_x \bar{C}, \\ B_1 &= \dot{b}_2 = -\dot{\bar{b}}_1, \quad B_2 = \partial_x b_2 = -\partial_x \bar{b}_1. \end{aligned} \quad (9)$$

Substituting the above relationships (9) into the super-expansion of the superfields [cf. (7)], we get following explicit expansions:

$$\begin{aligned}
\tilde{\mathcal{A}}_0^{(h)}(x, t, \theta, \bar{\theta}) &= A_0(x, t) + \theta \dot{C}(x, t) + \bar{\theta} \dot{C}(x, t) + i\theta\bar{\theta}\dot{b}(x, t), \\
&\equiv A_0(x, t) + \theta[s_b A_0(x, t)] + \bar{\theta}[s_{ab} A_0(x, t)] + \theta\bar{\theta}[s_b s_{ab} A_0(x, t)], \\
\tilde{\mathcal{A}}_1^{(h)}(x, t, \theta, \bar{\theta}) &= A_1(x, t) + \theta \bar{C}'(x, t) + \bar{\theta} C'(x, t) + i\theta\bar{\theta}b'(x, t), \\
&\equiv A_1(x, t) + \theta[s_b A_1(x, t)] + \bar{\theta}[s_{ab} A_1(x, t)] + \theta\bar{\theta}[s_b s_{ab} A_1(x, t)], \\
\mathcal{F}^{(h)}(x, t, \theta, \bar{\theta}) &= C(x, t) + \theta[-ib(x, t)], \\
&\equiv C(x, t) + \theta[s_b C(x, t)] + \bar{\theta}[s_{ab} C(x, t)] + \theta\bar{\theta}[s_b s_{ab} C(x, t)], \\
\bar{\mathcal{F}}^{(h)}(x, t, \theta, \bar{\theta}) &= \bar{C}(x, t) + \bar{\theta}[ib(x, t)] \\
&\equiv \bar{C}(x, t) + \theta[s_b \bar{C}(x, t)] + \bar{\theta}[s_{ab} \bar{C}(x, t)] + \theta\bar{\theta}[s_b s_{ab} \bar{C}(x, t)],
\end{aligned} \tag{10}$$

where we have chosen $b_2 = b = -\bar{b}_1$. The superscript (h) , in the above expression, denotes the superfields obtained after the application of HC. Thus, the (anti-)BRST symmetry transformations for the gauge field and (anti-)ghost fields can be easily inferred from the above expansions. Now, in order to derive (anti-)BRST symmetries for the other dynamical field (ϕ) and corresponding momenta (Π_ϕ) , we have to go beyond the BT superfield formalism. In this connection, it is to be noted that the following quantity:

$$A_1 - \frac{1}{e} \Pi_{(\phi)}, \tag{11}$$

remains invariant under the gauge transformations (4). This GIR serves our purpose in deriving the off-shell nilpotent (anti-) BRST transformations of the field $\Pi_{(\phi)}$. For this purpose we generalize the GIR in the superspace as follows:

$$\tilde{\mathcal{A}}_1^{(h)} - \frac{1}{e} \tilde{\Pi}_{(\phi)} = A_1 - \frac{1}{e} \Pi_{(\phi)} \tag{12}$$

where the superfield $\tilde{\Pi}_{(\phi)}$ is given by

$$\tilde{\Pi}_{(\phi)}(x, t, \theta, \bar{\theta}) = \Pi_{(\phi)}(x, t) + \theta \bar{f}_3(x, t) + \bar{\theta} f_3(x, t) + i\theta\bar{\theta} B_3(x, t), \tag{13}$$

where the secondary fields f_3, \bar{f}_3 are fermionic and B_3 is bosonic in nature. Exploiting (10), (12) and (13), we obtain

$$f_3 = e C', \quad \bar{f}_3 = e \bar{C}', \quad B_3 = e b'. \tag{14}$$

As a consequence, we can write

$$\begin{aligned}
\tilde{\Pi}_{(\phi)}^{(g,h)}(x, t, \theta, \bar{\theta}) &= \Pi_{(\phi)}(x, t) + \theta e \bar{C}'(x, t) + \bar{\theta} e C'(x, t) + i\theta\bar{\theta} e b'(x, t) \\
&\equiv \Pi_{(\phi)}(x, t) + \theta[s_b \Pi_{(\phi)}(x, t)] + \bar{\theta}[s_{ab} \Pi_{(\phi)}(x, t)] \\
&\quad + \theta\bar{\theta}[s_b s_{ab} \Pi_{(\phi)}(x, t)].
\end{aligned} \tag{15}$$

The (anti-)BRST symmetry transformations for the gauge and (anti-)ghost fields can be readily deduced from above expansions.

It is evident from (4) that the field $\phi(x, t)$ itself gauge invariant. As a consequence, the superfield $\tilde{\Phi}(x, t, \theta, \bar{\theta})$ corresponding to $\phi(x, t)$ has to be independent of the Grassmannian variables $(\theta, \bar{\theta})$. This statement can be corroborated in the following GIR as

$$\tilde{\Phi}(x, t, \theta, \bar{\theta}) = \phi(x, t). \quad (16)$$

Finally, we obtain the following off-shell nilpotent as well as absolutely anticommuting (anti-)BRST transformations for all fields:

$$\begin{aligned} s_b A_0 &= \dot{C}, \quad s_b A_1 = C', \quad s_b \Pi_{(\phi)} = eC', \quad s_b \bar{C} = ib, \quad s_b [b, C, \phi] = 0, \\ s_{ab} A_0 &= \dot{\bar{C}}, \quad s_{ab} A_1 = \bar{C}', \quad s_{ab} \Pi_{(\phi)} = e\bar{C}', \quad s_{ab} C = -ib, \quad s_{ab} [b, \bar{C}, \phi] = 0. \end{aligned} \quad (17)$$

The above (anti-)BRST transformations are off-shell nilpotent of order two (i.e. $s_{(a)b}^2 = 0$) and absolutely anticommuting (i.e. $s_b s_{ab} + s_{ab} s_b = 0$) in nature.

We point out that BRST transformation (s_b) of any generic field $\Omega(x, t)$ is equivalent to the translation of the corresponding superfield $\tilde{\Omega}(x, t, \theta, \bar{\theta})$ along the $\bar{\theta}$ -direction while keeping θ -direction fixed. Similarly, the anti-BRST transformation (s_{ab}) for any generic field can be obtain by taking the translation of superfield along the θ -direction and keeping $\bar{\theta}$ -direction intact. These statements can be mathematically corroborated as

$$s_b \Omega(x, t) = \partial_{\bar{\theta}} \tilde{\Omega}(x, t, \theta, \bar{\theta}) \Big|_{\bar{\theta}=0}, \quad s_{ab} \Omega(x, t) = \partial_{\theta} \tilde{\Omega}(x, t, \theta, \bar{\theta}) \Big|_{\theta=0}, \quad (18)$$

where the generic superfield is given in Eqs. (10), (15) and (16).

Using basic tenets of the BRST formulation, the gauge-fixed Lagrangian density (\mathcal{L}_b) which respects the above off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations can be written as

$$\begin{aligned} \mathcal{L}_b &= \mathcal{L}_f - s_b \left[i\bar{C} \left(\dot{A}_0 - A'_1 + \frac{1}{2} b \right) \right] \\ &\equiv \mathcal{L}_f + s_{ab} \left[i\bar{C} \left(\dot{A}_0 - A'_1 + \frac{1}{2} b \right) \right] \\ &\equiv \mathcal{L}_f + b(\dot{A}_0 - A'_1) + \frac{1}{2} b^2 - i\dot{\bar{C}}\dot{C} + i\bar{C}'C', \end{aligned} \quad (19)$$

where \mathcal{L}_f is the first order Lagrangian density given by (2) above.

4 (Anti-)co-BRST symmetries: superfield approach

In order to derive *proper* (i.e. off-shell nilpotent and absolutely anticommuting) (anti-)co-BRST symmetries, we shall take recourse to the dual horizontality condition (DHC) and the augmented version of superfield approach to BRST formalism. The DHC imposes following restriction [30]

$$\delta A^{(1)} = \tilde{\delta} \tilde{A}^{(1)}, \quad (20)$$

where $\delta(= - * d*)$ is the co-exterior derivative and $\tilde{\delta}(= - * \tilde{d}*)$ represents the super-co-exterior derivative. In other words, the above DHC implies that the gauge-fixing term,

which is invariant under the (anti-)co-BRST symmetries (see Ref. [25] for details), should remain unaffected by the presence of Grassmannian variables. Exploiting the above DHC (20) along with the superfield expansions defined in (7), we obtain

$$\partial_\theta F = 0, \quad \partial_{\bar{\theta}} \bar{F} = 0, \quad \partial_\theta \bar{F} + \partial_{\bar{\theta}} F = 0, \quad (21)$$

which, in turn, yields following relationships amongst the basic and secondary fields of the theory:

$$\bar{b}_1 = 0, \quad b_2 = 0, \quad s_1 = s_2 = 0, \quad b_1 = \mathcal{B} = -\bar{b}_2. \quad (22)$$

Substituting back these relationships into the superexpansion of the superfields (in (7)), we get

$$\begin{aligned} F^{(dh)}(x, t, \theta, \bar{\theta}) &= C(x, t) + \bar{\theta} [i\mathcal{B}(x, t)] \\ &\equiv C(x, t) + \theta(s_{ad}C(x, t)) + \bar{\theta}(s_d C(x, t)) + \theta\bar{\theta}(s_d s_{ad}C(x, t)), \\ \bar{F}^{(dh)}(x, t, \theta, \bar{\theta}) &= \bar{C}(x, t) + \theta [-i\mathcal{B}(x, t)] \\ &\equiv \bar{C}(x, t) + \theta(s_{ad}\bar{C}(x, t)) + \bar{\theta}(s_d \bar{C}(x, t)) + \theta\bar{\theta}(s_d s_{ad}\bar{C}(x, t)). \end{aligned} \quad (23)$$

Here the superscript (dh) represents the superexpansion after the application of DHC. From the above, we can identify the (anti-)co-BRST symmetries for the (anti-)ghost fields as listed below:

$$\begin{aligned} s_d C &= i\mathcal{B}, & s_d \bar{C} &= 0, & s_d s_{ad} C &= 0, \\ s_{ad} C &= 0, & s_{ad} \bar{C} &= -i\mathcal{B}, & s_{ad} s_{ad} \bar{C} &= 0. \end{aligned} \quad (24)$$

Now, we are free to choose the auxiliary field ‘ \mathcal{B} ’ in terms of basic fields of the theory so that both the crucial properties, i.e. the nilpotency and absolute anticommutativity, could be satisfied simultaneously. Thus, we choose $\mathcal{B} = (E - e\phi) \equiv (\dot{A}_1 - A'_0 - e\phi)$. We provide an explicit proof of this choice in Appendix A. However, with this choice of auxiliary field, in terms of basic fields of the theory, we can have only on-shell nilpotency. In order to restore the off-shell nilpotency, we linearize the kinetic term of the Lagrangian (cf. (19)) with the help of another auxiliary field, say \bar{b} , i.e. $(\frac{1}{2}E^2 \equiv \bar{b}E - \frac{1}{2}\bar{b}^2)$ (see Ref. [25] for details). Thus, using Euler-Lagrange equations of motion for \bar{b} , we get $\bar{b} = E$.

Therefore, keeping above in mind, Eq. (23) yields

$$\begin{aligned} F^{(dh)}(x, t, \theta, \bar{\theta}) &= C(x, t) + \theta(0) + \bar{\theta}[i(\bar{b} - e\phi)](x, t) + \theta\bar{\theta}(0), \\ \bar{F}^{(dh)}(x, t, \theta, \bar{\theta}) &= \bar{C}(x, t) + \theta[-i(\bar{b} - e\phi)](x, t) + \bar{\theta}(0) + \theta\bar{\theta}(0). \end{aligned} \quad (25)$$

Hence the (anti-)co-BRST symmetries for the (anti-)ghost fields can be easily identified as

$$\begin{aligned} s_d C &= i(\bar{b} - e\phi), & s_d \bar{b} &= 0, & s_d \bar{C} &= 0, & s_d s_{ad} C &= 0, \\ s_{ad} C &= 0, & s_{ad} \bar{C} &= -i(\bar{b} - e\phi), & s_{ad} \bar{b} &= 0, & s_{ad} s_{ad} \bar{C} &= 0, \end{aligned} \quad (26)$$

where $s_{(a)d} \bar{b} = 0$ is obtained due to the nilpotency property ($s_{(a)d}^2 = 0$). In order to derive the (anti-)co-BRST symmetries for the basic fields of the theory we have to go beyond the

DHC. Following from the symmetries of the Lagrangian density (2), we note that following quantities remain invariant under (anti-)co-BRST symmetries (see Ref. [25] for details):

$$\begin{aligned} s_{(a)d} [\dot{A}_0 - A'_1] &= 0, & s_{(a)d} [\Pi_{(\phi)} - eA_1] &= 0, \\ s_{(a)d} [\phi] &= 0, & s_{(a)d} [2e\Pi_{(\phi)}A_1 - \Pi_{(\phi)}^2 - e^2A_1^2] &= 0. \end{aligned} \quad (27)$$

These (anti-)co-BRST invariant quantities serve our purpose as they should remain independent of Grassmannian variables θ and $\bar{\theta}$ when the former are generalized onto the (2, 2)-dimensional supermanifold. Thus, we demand that

$$\begin{aligned} (\dot{\tilde{A}}_0 - \tilde{A}'_1)(x, t, \theta, \bar{\theta}) &= (\dot{A}_0 - A'_1)(x, t), \\ (\tilde{\Pi}_{(\phi)} - e\tilde{A}_1)(x, t, \theta, \bar{\theta}) &= (\Pi_{(\phi)} - eA_1)(x, t), \\ \tilde{\Phi}(x, t, \theta, \bar{\theta}) &= \phi(x, t). \end{aligned} \quad (28)$$

The first relation, in the above expression, along with (7), immediately implies

$$\dot{f}_1 - \bar{f}'_2 = 0, \quad \dot{f}_1 - f'_2 = 0, \quad \dot{B}_1 = B'_2. \quad (29)$$

Here we make the judicious choice, guided by the basic ingredients of augmented superfield formalism, as $f_1 = -\bar{C}'$, $\bar{f}_1 = -C'$, $f_2 = -\dot{\bar{C}}$, $\bar{f}_2 = -\dot{C}$. However, we provide a precise proof of this choice in Appendix A. With these choices, the superexpansion of superfields (cf. (7)) can be written as,

$$\begin{aligned} \tilde{A}_0^{(as)}(x, t, \theta, \bar{\theta}) &= A_0(x, t) - \theta C'(x, t) - \bar{\theta} \bar{C}'(x, t) - i\theta\bar{\theta}(\bar{b}' - e\phi')(x, t), \\ \tilde{A}_1^{(as)}(x, t, \theta, \bar{\theta}) &= A_1(x, t) - \theta \dot{C}(x, t) - \bar{\theta} \dot{\bar{C}}(x, t) - i\theta\bar{\theta}(\dot{\bar{b}} - e\dot{\phi})(x, t), \end{aligned} \quad (30)$$

where the superscript (as) represents the expansions after the application of the augmented superfield formalism. The second relation in (28) along with the inputs from (13) implies

$$\bar{f}_3 = e\bar{f}_2, \quad f_3 = ef_2, \quad B_3 = eB_2, \quad (31)$$

which, with the help of (29) and (30), yields

$$\bar{f}_3 = -e\dot{C}, \quad f_3 = -e\dot{\bar{C}}, \quad B_3 = -e(\dot{\bar{b}} - e\dot{\phi}). \quad (32)$$

Thus, Eq. (13) reduces to

$$\begin{aligned} \tilde{\Pi}_{(\phi)}^{(as)}(x, t, \theta, \bar{\theta}) &= \Pi_{(\phi)}(x, t) - \theta(e\dot{C})(x, t) - \bar{\theta}(e\dot{\bar{C}})(x, t) - i\theta\bar{\theta}e(\dot{\bar{b}} - e\dot{\phi})(x, t) \\ &\equiv \Pi_{(\phi)}(x, t) + \theta[s_d\Pi_{(\phi)}(x, t)] + \bar{\theta}[s_{ad}\Pi_{(\phi)}(x, t)] \\ &\quad + \theta\bar{\theta}[s_ds_{ad}\Pi_{(\phi)}(x, t)]. \end{aligned} \quad (33)$$

As, it clear from the (27) that the field $\phi(x, t)$ remains invariant under (anti-)co-BRST symmetry transformation. As a consequence, the (anti-)co-BRST symmetries can be deduced trivially. Thus, we obtain the following nilpotent as well as absolutely anticommuting (anti-)co-BRST transformations for all fields:

$$\begin{aligned} s_d A_0 &= -\bar{C}', & s_d A_1 &= -\dot{\bar{C}}, & s_d \Pi_{(\phi)} &= -e\dot{\bar{C}}, \\ s_d C &= i(\bar{b} - e\phi), & s_d [\bar{C}, b, \bar{b}, \phi] &= 0, \\ s_{ad} A_0 &= -C', & s_{ad} A_1 &= -\dot{C}, & s_{ad} \Pi_{(\phi)} &= -e\dot{C}, \\ s_{ad} \bar{C} &= -i(\bar{b} - e\phi), & s_{ad} [C, b, \bar{b}, \phi] &= 0. \end{aligned} \quad (34)$$

The above (anti-)co-BRST transformations are on-shell nilpotent of order two (i.e. $s_{(a)d}^2 = 0$) and absolutely anticommuting in nature (i.e. $s_d s_{ad} + s_{ad} s_d = 0$) in nature.

At this juncture, it is worthwhile to mention that co-BRST transformation of any generic field $\Omega(x, t)$ is equivalent to the translation of the corresponding superfield $\Omega(x, t, \theta, \bar{\theta})$ along the $\bar{\theta}$ -direction while keeping θ -direction fixed. Similarly, the anti-co-BRST transformation of any generic field can be obtain by taking the translational of the superfield along the θ -direction and keeping $\bar{\theta}$ -direction intact. Mathematically,

$$s_d \Omega(x, t) = \partial_{\bar{\theta}} \Omega(x, t, \theta, \bar{\theta}) \Big|_{\bar{\theta}=0}, \quad s_{ad} \Omega(x, t) = \partial_{\theta} \Omega(x, t, \theta, \bar{\theta}) \Big|_{\bar{\theta}=0}, \quad (35)$$

where the generic supefields are given in Eqs. (23), (30) and (33).

5 Nilpotency and absolute anticommutativity: Super-field approach

In this section, we capture the nilpotency and absolute anticommutativity property of the (anti-)BRST as well as (anti-)co-BRST charges within the framework of superfield formalism. Exploiting the standard techniques of the Noether theorem, it is easy to check that the (anti-)BRST symmetries (cf. (17)) lead to the derivation of following nilpotent ($Q_{(a)b}^2 = 0$) and conserved (anti-)BRST charges $Q_{(a)b}$ as listed below:

$$Q_{ab} = \int dx [b \dot{\bar{C}} + E \bar{C}' + e \phi' \bar{C}] \quad Q_b = \int dx [b \dot{C} + E C' + e \phi' C]. \quad (36)$$

The conservation law of above (anti-)BRST charges can be proven with the help of following Euler-Lagrange equations of motion

$$\begin{aligned} \square C &= 0, & \Pi_{\phi} &= \dot{\phi} + e A_1, & E' &= \dot{b} + e \phi', \\ \square \bar{C} &= 0, & \dot{E} &= b' + e(\Pi_{\phi} - e A_1), & \dot{\Pi}_{\phi} &= \phi'' + e A_0'. \end{aligned} \quad (37)$$

At this juncture, it is interesting to note that the (anti-)BRST charges (cf. (36)) can also be written, up to a surface term, in the following form with the help of above mentioned Euler-Lagrange equations of motion:

$$Q_{ab} = \int dx [b \dot{\bar{C}} - \dot{b} \bar{C}], \quad Q_b = \int dx [b \dot{C} - \dot{b} C]. \quad (38)$$

It is straightforward to check, with help of (17), that the following is true:

$$\begin{aligned} Q_{ab} &= \int dx s_{ab} [i (\bar{C} \dot{C} - \dot{\bar{C}} C)] \equiv \int dx [s_b (i \dot{\bar{C}} \bar{C})], \\ Q_b &= \int dx s_b [-i (\bar{C} \dot{C} - \dot{\bar{C}} C)] \equiv \int dx [s_{ab} (-i \dot{C} C)]. \end{aligned} \quad (39)$$

The above (anti-)BRST charges can be expressed in terms of superfields as follows:

$$\begin{aligned}
Q_{ab} &= i \int dx \left[\frac{\partial}{\partial \theta} \left(\bar{F}^{(h)}(x, \theta, \bar{\theta}) \dot{F}^{(h)}(x, \theta, \bar{\theta}) - \dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\bar{\theta}=0} \\
&\equiv i \int dx \left[\int d\theta \left(\bar{F}^{(h)}(x, \theta, \bar{\theta}) \dot{F}^{(h)}(x, \theta, \bar{\theta}) - \dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\bar{\theta}=0}, \\
Q_{ab} &= i \int dx \left[\frac{\partial}{\partial \bar{\theta}} \left(\dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) \bar{F}^{(h)}(x, \theta, \bar{\theta}) \right) \right] \\
&\equiv i \int dx \left[\int d\bar{\theta} \left(\dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) \bar{F}^{(h)}(x, \theta, \bar{\theta}) \right) \right], \\
Q_b &= -i \int dx \left[\frac{\partial}{\partial \theta} \left(\bar{F}^{(h)}(x, \theta, \bar{\theta}) \dot{F}^{(h)}(x, \theta, \bar{\theta}) - \dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\theta=0} \\
&\equiv -i \int dx \left[\int d\bar{\theta} \left(\bar{F}^{(h)}(x, \theta, \bar{\theta}) \dot{F}^{(h)}(x, \theta, \bar{\theta}) - \dot{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\theta=0}, \\
Q_b &= -i \int dx \left[\frac{\partial}{\partial \theta} \left(\dot{F}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right], \\
&\equiv -i \int dx \left[\int d\theta \left(\dot{F}^{(h)}(x, \theta, \bar{\theta}) F^{(h)}(x, \theta, \bar{\theta}) \right) \right]. \tag{40}
\end{aligned}$$

From the above expressions it is clear that $\partial_{\bar{\theta}} Q_b = 0$, $\partial_{\theta} Q_{ab} = 0$ hold because of the nilpotency properties (i.e. $\partial_{\bar{\theta}}^2 = 0$, $\partial_{\theta}^2 = 0$) of the translational generators $\partial_{\bar{\theta}}$, ∂_{θ} , respectively. This, in turn, implies that the nilpotency ($Q_{(a)b}^2 = 0$) of (anti-)BRST charges is encoded in the following observation

$$\begin{aligned}
\partial_{\theta} Q_{ab} = 0 &\iff s_{ab} Q_{ab} = i \{Q_{ab}, Q_{ab}\} = 0 \implies Q_{ab}^2 = 0, \\
\partial_{\bar{\theta}} Q_b = 0 &\iff s_b Q_b = i \{Q_b, Q_b\} = 0 \implies Q_b^2 = 0. \tag{41}
\end{aligned}$$

At this juncture, we would like to point out that $\partial_{\theta} Q_b = 0$ and $\partial_{\bar{\theta}} Q_{ab} = 0$ is also true, which can be translated in the language of (anti-)BRST symmetry transformations as follows:

$$\begin{aligned}
\partial_{\bar{\theta}} Q_{ab} = 0 &\iff s_b Q_{ab} = i \{Q_{ab}, Q_b\} = 0 \implies Q_{ab} Q_b + Q_b Q_{ab} = 0, \\
\partial_{\theta} Q_b = 0 &\iff s_{ab} Q_b = i \{Q_b, Q_{ab}\} = 0 \implies Q_b Q_{ab} + Q_{ab} Q_b = 0. \tag{42}
\end{aligned}$$

Thus, we have been able to capture the nilpotency and absolute anticommutativity of (anti-)BRST charges in the language of augmented superfield formalism.

Similarly, we can also capture the nilpotency and absolute anticommutativity of the (anti-)co-BRST charges. For this purpose, we start with the (anti-)co-BRST charges, which can be expressed as

$$\begin{aligned}
Q_{ad} &= - \int dx [(\bar{b} - e\phi)\dot{C} + bC'] \equiv - \int dx [(\bar{b} - e\phi)\dot{C} - (\dot{\bar{b}} - e\dot{\phi})C], \\
Q_d &= - \int dx [(\bar{b} - e\phi)\dot{\bar{C}} + b\bar{C}'] \equiv - \int dx [(\bar{b} - e\phi)\dot{\bar{C}} - (\dot{\bar{b}} - e\dot{\phi})\bar{C}]. \tag{43}
\end{aligned}$$

These (anti-)co-BRST charges can also be written in the following more convenient way such that the nilpotency and absolute anticommutativity becomes clear and easy to check:

$$\begin{aligned} Q_{ad} &= i \int dx s_{ad} [C \dot{\bar{C}} + \bar{C} \dot{C}] \equiv -i \int dx s_d (\dot{C} C), \\ Q_d &= i \int dx s_d [C \dot{\bar{C}} + \bar{C} \dot{C}] \equiv i \int dx s_{ad} (\dot{\bar{C}} \bar{C}). \end{aligned} \quad (44)$$

In terms of superfield, within the framework of augmented superfield formalism, these (anti-)co-BRST charges can also be written in the following manner:

$$\begin{aligned} Q_{ad} &= \int dx \left[\frac{\partial}{\partial \theta} \left(i \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \dot{F}^{(dh)}(x, \theta, \bar{\theta}) - i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\bar{\theta}=0} \\ &\equiv \int dx \left[\int d\theta \left(i \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \dot{F}^{(dh)}(x, \theta, \bar{\theta}) - i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\bar{\theta}=0}, \\ Q_{ad} &= -i \int dx \left[\frac{\partial}{\partial \theta} \left(\dot{F}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \\ &\equiv -i \int dx \left[\int d\bar{\theta} \left(\dot{F}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right], \\ Q_d &= \int dx \left[\frac{\partial}{\partial \theta} \left(i \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \dot{F}^{(dh)}(x, \theta, \bar{\theta}) - i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\theta=0} \\ &\equiv \int dx \left[\int d\bar{\theta} \left(i \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \dot{F}^{(dh)}(x, \theta, \bar{\theta}) - i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) F^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \Big|_{\theta=0}, \\ Q_d &= \int dx \left[\frac{\partial}{\partial \theta} \left(i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \right) \right] \\ &\equiv \int dx \left[\int d\theta \left(i \dot{\bar{F}}^{(dh)}(x, \theta, \bar{\theta}) \bar{F}^{(dh)}(x, \theta, \bar{\theta}) \right) \right]. \end{aligned} \quad (45)$$

The nilpotency and absolute anticommutativity of (anti-)co-BRST charges are straightforward to check. For instance

$$\begin{aligned} \partial_\theta Q_{ad} = 0 &\iff s_{ad} Q_{ad} = i \{Q_{ad}, Q_{ad}\} \implies Q_{ad}^2 = 0, \\ \partial_{\bar{\theta}} Q_d = 0 &\iff s_d Q_d = i \{Q_d, Q_d\} \implies Q_d^2 = 0, \end{aligned} \quad (46)$$

is true because of the nilpotency (i.e. $\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$) of the translation generators. This precisely proves the nilpotency ($Q_{(a)d}^2 = 0$) of the (anti-)co-BRST charges. For the proof of anticommutativity, it is interesting to note that following expressions also hold, namely,

$$\begin{aligned} \partial_\theta Q_d = 0 &\iff s_{ad} Q_d = i \{Q_d, Q_{ad}\} = 0 \implies Q_d Q_{ad} + Q_{ad} Q_d = 0, \\ \partial_{\bar{\theta}} Q_{ad} = 0 &\iff s_d Q_{ad} = i \{Q_{ad}, Q_d\} = 0 \implies Q_{ad} Q_d + Q_d Q_{ad} = 0, \end{aligned} \quad (47)$$

due to nilpotency ($\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$) and absolute anticommutativity properties ($\partial_\theta \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_\theta = 0$) of the translational generators. This clearly demonstrate the absolute anticommutativity properties of (anti-)co-BRST charges within the framework of augmented superfield formalism. Thus, we conclude that the nilpotency and absolute anticommutativity properties of (anti-)BRST as well as (anti-)co-BRST charges are connected with such properties of translational generators along the Grassmannian directions of the $(2, 2)$ -dimensional supermanifold.

6 Conclusions

The central results of our present investigation are the precise derivation of the *proper* (i.e. nilpotent and absolutely anticommuting) (anti-)BRST as well as (anti-)co-BRST symmetry transformations for the 2D bosonized version of vector Schwinger model within the framework of augmented superfield formalism. We have made use of the horizontality condition (HC) and gauge invariant restriction to derive the (anti-)BRST symmetries for all the fields of the underlying theory. Additionally, in order to derive the complete set of (anti-)co-BRST symmetries, for all the fields of the present theory, we have exploited the power and strength of dual-HC condition and (anti-)co-BRST invariant restrictions.

We have provided the geometrical origin of the above mentioned continuous symmetries (and their corresponding generators) in the language of translation generators along the Grassmannian directions of the $(2, 2)$ -dimensional supermanifold, on which VSM is generalized. Furthermore, within the framework of augmented superfield formalism, we expressed the (anti-)BRST and (anti-)co-BRST charges in various forms. Subsequently, we have been able to capture the nilpotency and absolute anticommutativity of above mentioned charges in the framework of superfield formalism. We have also shown that the nilpotency and absolute anticommutativity are connected with such properties of translational generators along the Grassmannian directions of the $(2, 2)$ -dimensional supermanifold.

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Appendix A. On the specific choice of auxiliary field

In this appendix, we provide explicit derivation of the ad hoc choice made in Sec. 4 for the auxiliary field

$$\mathcal{B} = E - e\phi \equiv \dot{A}_1 - A'_0 - e\phi, \quad (48)$$

which ensues the relations $B_1 = -i(\bar{b}' - e\phi')$, $B_2 = -i(\bar{b} - e\dot{\phi})$ and also connected with the choices $f_1 = -\bar{C}'$, $\bar{f}_1 = -C'$, $f_2 = -\bar{C}$, $\bar{f}_2 = -\dot{C}$ (cf. comment after (29)). This choice is guided by the basic ingredients of superfield formalism according to which all the (anti-)BRST (and/or (anti-)co-BRST) invariant quantities should remain independent of the Grassmannian variables θ and $\bar{\theta}$ when former are generalized on to the supermanifold. Following the above logic, we note that the following quantities:

$$K_1 = A_1\dot{\bar{C}}, \quad K_2 = A_1\dot{C}, \quad K_3 = A_0\bar{C}', \quad K_4 = A_0C', \quad (49)$$

remain invariant under (anti-)co-BRST symmetry transformations (i.e. $s_d K_1 = 0$, $s_{ad} K_2 = 0$, $s_d K_3 = 0$, $s_{ad} K_4 = 0$). Thus, keeping above in mind, we demand that

$$\begin{aligned}\tilde{\mathcal{A}}_1(x, t, \theta, \bar{\theta}) \dot{\bar{F}}^{(dh)}(x, t, \theta, \bar{\theta}) &= A_1(x, t) \dot{\bar{C}}(x, t), \\ \tilde{\mathcal{A}}_1(x, t, \theta, \bar{\theta}) \dot{F}^{(dh)}(x, t, \theta, \bar{\theta}) &= A_1(x, t) \dot{C}(x, t), \\ \tilde{\mathcal{A}}_0(x, t, \theta, \bar{\theta}) \bar{F}'^{(dh)}(x, t, \theta, \bar{\theta}) &= A_0(x, t) \bar{C}'(x, t), \\ \tilde{\mathcal{A}}_0(x, t, \theta, \bar{\theta}) F'^{(dh)}(x, t, \theta, \bar{\theta}) &= A_0(x, t) C'(x, t),\end{aligned}\tag{50}$$

which lead to the following relationships:

$$\begin{aligned}\bar{f}_2 \dot{\bar{C}} - i A_1 \dot{\mathcal{B}} &= 0, & f_2 \dot{\bar{C}} &= 0, & f_2 \dot{\mathcal{B}} - B_2 \dot{\bar{C}} &= 0, \\ \bar{f}_2 \dot{C} + i A_1 \dot{\mathcal{B}} &= 0, & \bar{f}_2 \dot{C} &= 0, & \bar{f}_2 \dot{\mathcal{B}} - B_2 \dot{C} &= 0, \\ \bar{f}_1 \bar{C}' - i A_0 \mathcal{B}' &= 0, & f_1 \bar{C}' &= 0, & f_1 \mathcal{B}' - B_1 \bar{C}' &= 0, \\ \bar{f}_1 C' + i A_0 \mathcal{B}' &= 0, & \bar{f}_1 C' &= 0, & \bar{f}_1 \mathcal{B}' - B_1 C' &= 0.\end{aligned}\tag{51}$$

These relationships immediately imply that $f_1 \propto \bar{C}'$, $\bar{f}_1 \propto C'$, $f_2 \propto \dot{\bar{C}}$, $\bar{f}_2 \propto \dot{C}$, $B_1 \propto \mathcal{B}'$, $B_2 \propto \dot{\mathcal{B}}$. We have made similar choices (cf. Sec. 4) with a minus sign for algebraic convenience. Thus, we have derived all the secondary fields in term of basic fields of the theory which leads to the derivation of full set of (anti-)co-BRST symmetries.

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